Citation for published version:
Brampton, CJ, Betts, DN, Bowen, CR & Kim, HA 2013, 'Sensitivity of bistable laminates to uncertainties in material properties, geometry and environmental conditions', *Composite Structures*, vol. 102, pp. 276-286. https://doi.org/10.1016/j.compstruct.2013.03.005

DOI:
10.1016/j.compstruct.2013.03.005

Publication date:
2013

Document Version
Peer reviewed version

Link to publication

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Sensitivity of bistable laminates to uncertainties in material properties, geometry and environmental conditions

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Keywords: Composites, Laminates, Asymmetric, Bistable, Modelling, Sensitivity

ABSTRACT

Under certain conditions asymmetric composite laminates can have a bistable response to mechanical loading. A transition between the two stable states provides opportunities to produce large deflections or shape changes from relatively low energy inputs that do not need to be maintained to sustain a specific shape. Such laminates are attracting interest in aerospace applications, deployable structures and energy harvesting. Accurate modelling predictions of bistable laminate shapes has proven challenging, in part due to uncertainties in geometry, material properties and the operating environment of the laminates. In this paper a detailed sensitivity analysis of the influence of each of these properties on laminate curvature is undertaken and demonstrates that bistable laminates are most sensitive to uncertainties in the Young's moduli, thermal expansion coefficients, ply thickness and the temperature change from the elevated cure temperature. Accurate characterisation of these properties and quality control during manufacture can reduce the discrepancies between analytical models and experimental results and allow the models to be used as viable tools for the design of bistable laminates. It is also shown that laminates are highly sensitive to moisture absorption and temperature changes, especially when changes in material properties due to temperature were included in the modelling.

1. Introduction

Bistable structures have two stable shapes which only require energy to induce a transition between the stable states, and no continuous energy input is required to maintain one particular state. This is a common property of asymmetric composite laminates of specific lay-up and geometry [1]. In laminates, providing specific geometric conditions are met, bistability is induced by the anisotropic thermal expansion of the individual layers on cooling from the high curing temperatures during manufacture [1]. The key geometric factor is the edge length to thickness ratio and figure 1 shows the relationship between this ratio and the curvature for a square $[0/90]_T$ laminate. At low edge length to thickness ratios only a single stable saddle shape exists, point A. However beyond a critical bifurcation thickness ratio, point B, the laminate develops two stable cylindrical shapes, points C and D, with an unstable saddle shape also predicted by modelling (dashed line). The location of this bifurcation point varies with laminate geometry, stacking sequence, material properties and environmental conditions.

Transforming a bistable laminate from one stable shape to another using mechanical loads or smart actuators can create a large displacement from a relatively small energy input. This is an attractive property for aerospace and deployable structure applications [2] where the use of bistable laminates is being considered [3]. Bistable laminates have also recently attracted interest in piezoelectric energy harvesting applications [4-6].

However challenges still remain for bistable laminates to be used in many applications; one of which is the difficulty in accurately modelling composite laminate shapes, and the conditions under which bistability exists. Numerous studies to date have used finite element and analytical methods to model bistable composite shapes and transformations [7-16]. The use of bistable spring networks has also been proposed to model bistable composite laminates, as has been successfully used to model the multi-stable behaviour of other smart and biological materials [17-23].

The complexity of analytical methods has been increased over time to include in-plane shear strains [7] and increasingly complex polynomials to approximate the displacements [9] and the midplane strains [10]. With these improvements analytical models have been shown to produce similar results to finite element models [10, 30]; however discrepancies between modelling and experimental results are still observed [9, 10, 12, 16]. For example a recent study by Betts et al [12] found up to a 7.2% difference in out-of-plane displacement between analytical and experimental results. However, while some of this inaccuracy can be traced to simplified modelling assumptions commonly used in the analytical modelling, such as the assumption of zero inter-laminar stress on the free-edge deformation of the laminate [12, 24], it is hypothesised here that a significant proportion of the error between experiment and prediction originates from uncertainties in material properties, manufacturing and environmental conditions that are often not considered in the analytical models.

The numerical models accurately predict the shapes of an idealised or 'perfect' laminate, while experiments are performed on imperfect laminates operating in variable environments. As a result, an improved understanding of how these uncertainties affect the response of asymmetric composite laminates will aid in producing more accurate numerical models of bistable composites. Furthermore, the most influential parameters requiring the greatest degree of control can be identified and quantified.

The first source of uncertainty considered here is variability in the composite material properties which can occur for two reasons. Firstly some material properties required for modelling are difficult to accurately measure, in particular Poisson's ratio [26]. Secondly, the pre-preg material properties can change over time under storage [27]. As an example, a typical pre-preg material will have a useful shelf life of around 30 days at room temperature, prior to curing [27].

The second source of uncertainty is the geometry of the laminate. The idealised model assumes that each individual ply has a uniform geometry while manufactured plies can have geometric imperfections that have a significant effect on the laminate shape, with the ply thickness being of particular importance [25]. Hamamoto and Hyer [25] found that a variation in the layer thickness as little as 1% can have a significant effect on the temperature-curvature relationship of the laminate and can eliminate bistability. Optical microscopy analysis of laminate cross-sections suggest that there is typically a ±2% variation in individual ply thickness from manufacturing, with the variation being noticeably higher in plies on the exposed surface during manufacture [12]. Additionally, poor manufacturing control can lead to a thin layer of resin curing on the upper surface of the laminate [16]. Previous models of this variation in laminate geometry have shown that it can lead to a 4.6% variation in the out-of-plane displacement of a bistable laminate, a significant proportion of the 7.6% overall error in the displacement calculated by the analytical model [12].

The final source of uncertainty to be considered is the environmental conditions experienced by the laminate during both manufacture and operation. The effect of temperature upon the curvature of bistable laminates is well documented [12] as the anisotropic thermal expansion of the composite layers is the source of bistability. Laminate curvature would be expected to increase as the difference between the operating temperature and cure temperature increases. This introduces two sources of uncertainty, as variation in both the cure and ambient temperature will affect the curvature. While it is possible to exert quality control over the cure temperature during manufacture, control of the ambient temperature can be difficult and is impractical in many real applications. It is thus particularly important to be able to predict how the shapes of a laminate are affected by changes in temperature for aerospace applications where large variations in temperature are to be expected, with temperatures up to 40°C on the ground to as low as -60°C during cruise [3].

The influence of temperature on the material properties is an additional effect to consider. Hyer et al [26] published detailed experimental measurements of the variation of the material properties of a T300/5028 graphite-epoxy laminate with temperature. Moore et al [28] recently modelled bistable laminates using this data and demonstrated that it led to a non-linear relationship between laminate curvature and temperature; such a response is significantly different from the linear relationship predicted assuming constant material properties [16]. The temperature dependence of properties is often ignored in modelling of bistable laminates [5-13].

Other environmental effects such as moisture content can also be significant. It is widely known that the epoxy resin systems used in the manufacture of laminates absorb atmospheric moisture [29]. This leads to an expansion of the laminate layer which is proportional to the moisture expansion coefficients that relaxes the residual stresses induced by the anisotropic thermal contraction [14, 15] and reduces the curvature. It is also theoretically possible that sufficient moisture absorption could induce monostability in a laminate; although to date this has not been observed in experimental conditions [15].

Moisture absorption has been considered by Portela et al [14] and Etches et al [15] using both finite element and analytical modelling methods. Etches et al [15] modelled the effect for a range of relative humidities and compared predictions with experimental results for multiple samples. It was shown that the change in curvature with moisture absorption is small for relative humidities of approximately 40% or less (corresponding to 0.6wt% moisture content) with the variation becoming more pronounced beyond 40%. In any application where exposure to high humidity for extended periods is possible, the effect of moisture on the laminate shapes should be considered.

The aim of this paper is therefore to quantify the sensitivity of the curvatures of bistable composite laminates to uncertainties in the parameters described above. This is to include consideration of the material properties, laminate geometry and operating environment (temperature and moisture content). It is intended to identify which properties have the greatest effect on laminate shape. Identification of the important properties would allow refinement of manufacturing processes to increase control over key properties and to identify the properties that require precise characterisation for accurate numerical modelling. The importance of considering the variation in environmental operating conditions during laminate design will also be considered.

2. Analytical modelling method

In this paper the analytical model introduced by Dano and Hyer [10] is used to calculate the shapes of asymmetric bistable laminates. The model is a non-linear extension of classical laminated plate theory (CLT) using non-zero in-plane shear strains and approximated mid-plane strain functions. The shape is defined at a given temperature by finding the minimum strain energy for the laminate (and hence the stable shape) using a Rayleigh-Ritz minimisation. This model has been previously compared against finite elements models [10, 11, 30] and has been used in numerous previous experimental investigations [10, 12, 30]; therefore only a brief outline of the approach is provided here. We extend the model to include the moisture expansion terms, as defined by Portela et al. [14].

The coordinate system used to define the laminate is defined with x and y as the in-plane directions and z as the out-of-plane direction with the origin at the centre of the laminate. The out-of-plane displacement (w) is assumed to be of the following form:

$$w(x,y) = 0.5(ax^2 + by^2 + cxy) \qquad (1)$$

where a, b and c are equal to the negative values of the curvatures κ_x, κ_y and κ_{xy} respectively. The mid-plane strains are approximated using third order polynomials. Dano and Hyer [10] considered the complete polynomials and demonstrated that the coefficients of the terms with powers of x and y which sum to odd numbers are always zero [10], therefore the following forms are used.

$$\varepsilon_x^0 = \frac{du^0}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 = d_1 + d_2x^2 + d_3xy + d_4y^2$$

$$\varepsilon_y^0 = \frac{dv^0}{dy} + \frac{1}{2}\left(\frac{dw}{dy}\right)^2 = d_5 + d_6x^2 + d_7xy + d_8y^2 \qquad (2)$$

$$\varepsilon_{xy}^0 = \frac{du^0}{dy} + \frac{dv^0}{dx} + \frac{dw}{dx}\frac{dw}{dy}$$

Where u^0 and v^0 are the in-plane displacements in the x- and y- directions respectively. By combining Eqs. (1) and (2) and introducing integration constants d_{9-11} the equations for the in-plane displacements can be defined:

$$u^0(x,y) = d_1x + d_9y + \frac{1}{2}\left(d_3 - \frac{1}{2}ac\right)x^2y + \left(d_4 - \frac{c^2}{8}\right)xy^2 + \frac{1}{3}\left(d_2 - \frac{1}{2}a^2\right)x^3 + \frac{1}{3}d_{11}y^3 \qquad (3)$$

$$v^0(x,y) = d_9x + d_5y + \frac{1}{2}\left(d_7 - \frac{1}{2}bc\right)xy^2 + \left(d_6 - \frac{c^2}{8}\right)x^2y + \frac{1}{3}\left(d_8 - \frac{1}{2}b^2\right)y^3 + \frac{1}{3}d_{10}x^3$$

The total strain energy, W, of the laminate can be expressed as the integral of the strain energy density, ω, over the laminate volume:

$W = \int_{vol} \omega d(vol)$ (4)

$\omega = \frac{1}{2} c_{ijkl}\varepsilon_{ij}\varepsilon_{kl} - \hat{\alpha}_{ij}\varepsilon_{ij}\Delta T + m\varepsilon_{ij}\hat{\beta}_{ij}$ (5)

Where c_{ijkl} are the elastic constants, ε_{ij} and ε_{kl} are the total strains, $\hat{\alpha}_{ij}$ are constants related to the thermal expansion coefficients, ΔT is the change in temperature, m is the moisture content of the laminate as a percentage of laminate weight and $\hat{\beta}_{ij}$ are constants related to the coefficients of moisture expansion. CLT follows the assumption that the through-thickness stresses are small in comparison to in-plane stresses. In assuming a state of plane stress, the normal stress in the *z*-direction and out-of-plane shear stress are assumed to be zero. Expansion of Eq. (4) then leads to the following expression for the total strain energy,

$$W = \int_{-L_x/2}^{L_x/2} \int_{-L_y/2}^{L_y/2} \int_{-H/2}^{H/2} \frac{1}{2}\bar{Q}_{11}{\varepsilon_x}^2 + \bar{Q}_{12}\varepsilon_x\varepsilon_y + \bar{Q}_{16}\varepsilon_{xy}\varepsilon_y + \frac{1}{2}\bar{Q}_{22}{\varepsilon_y}^2 + \bar{Q}_{26}\varepsilon_{xy}\varepsilon_y + \frac{1}{2}\bar{Q}_{66}{\varepsilon_{xy}}^2$$
$$- (\bar{Q}_{11}\alpha_x + \bar{Q}_{12}\alpha_y + \bar{Q}_{16}\alpha_{xy})\varepsilon_x\Delta T - (\bar{Q}_{12}\alpha_x + \bar{Q}_{22}\alpha_y + \bar{Q}_{26}\alpha_{xy})\varepsilon_y\Delta T$$
$$- (\bar{Q}_{16}\alpha_x + \bar{Q}_{26}\alpha_y + \bar{Q}_{66}\alpha_{xy})\varepsilon_{xy}\Delta T - (\bar{Q}_{11}\beta_x + \bar{Q}_{12}\beta_y + \bar{Q}_{16}\beta_{xy})\varepsilon_x m$$
$$- (\bar{Q}_{12}\beta_x + \bar{Q}_{22}\beta_y + \bar{Q}_{26}\beta_{xy})\varepsilon_y m - (\bar{Q}_{16}\beta_x + \bar{Q}_{26}\beta_y + \bar{Q}_{66}\beta_{xy})\varepsilon_{xy} m dxdydz$$

(6)

Where the $\bar{Q}_{ij}'s$ are the symmetric transformed stiffness matrices for each of the individual layers, L_x and L_y the edge lengths of the laminate, H is the total laminate thickness (the sum of the individual ply thickness t) and the total strains are defined as,

$$\varepsilon_x = \varepsilon_x^0 - za$$
$$\varepsilon_y = \varepsilon_y^0 - zb \quad (7)$$
$$\varepsilon_{xy} = \varepsilon_{xy}^0 - zc$$

Substitution of Eq. (7) into Eq. (6), and the subsequent spatial integration of Eq. (6) leads to an expression for the total energy of the laminate of the following form:

$W = W(a, b, c, d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{10}, d_{11})$ (8)

The minimum strain energy states are then found by finding points where:

$dW = \sum_{i=1}^{14} \frac{dW}{dx_i} dx_i = 0$ (9)

Where x_i's are $a, b, c, d_1 \dots d_{11}$. The stable shapes are defined by the 14 coefficients x_i which can be found by solving the 14 nonlinear equations defined below.

$f_i = \frac{dW}{dx_i} = 0; \qquad i = 1,2 \dots 14$ (10)

In this form it is noted that throughout the 14 equations the shape coefficients d_{1-11} appear independently as linear terms only. However the equations are not linear with respect to the out-of-plane coefficients a, b and c. The system can therefore be reduced to just three equations and the three unknown curvatures a, b and c. This process results in a system defining the equilibrium position of the following form, which can be solved efficiently using numerical solution methods.

$$\frac{dW(a,b,c)}{da} = \frac{dW(a,b,c)}{db} = \frac{dW(a,b,c)}{dc} = 0 \qquad (11)$$

2.2 Material properties and their temperature dependency

The material properties used for this investigation are those determined by Hyer et al [26] for a T300/5028 graphite-epoxy composite. These properties were used as they were determined experimentally within an environmental chamber and their temperature dependency has been characterised over a wide temperature range (-150°C to 120°C). Hyer et al [26] found significant variation in the material properties with temperature with the exception of the transverse thermal expansion coefficient (α_{22}), which remained approximately constant. Polynomials to describe the variation of each property with temperature were derived from the lines fitted to the experimental data. The coefficient of moisture expansion was calculated using the room temperature value for T300/5028 graphite-epoxy composite from Portela et al [14]. The values and temperature dependent formulation of the material properties are shown in Table 1. Initially the room temperature values will be used in modelling to determine which materials properties have a significant influence on laminate curvature; this will identify which properties should be characterised accurately for modelling purposes. The temperature dependent properties are considered later in the paper to identify which properties are highly variable with temperature and also influence laminate curvature and should therefore be also characterised as a function of temperature.

3. Modelling sensitivity analysis

In this section the sensitivity of the laminate shapes to each of the individual uncertainties will be considered. A single square $[0/90]_T$ cross-ply laminate is modelled for edge length to thickness ratios of $2 \leq (L/t) \leq 250$, using the analytical method described in section 2 and the room temperature material properties in Table 1. The change in *x*-curvature with geometry for this $[0/90]_T$ cross-ply laminate is shown in Fig. 2, providing a baseline value for comparison. Alternative stacking sequences will also be discussed. The model is re-run for a ±5% change in each property where the uncertainty is split into 101 increments. The sensitivity of the laminate shape to each property is expressed by the change in principal curvature (*x*-curvature, *a*) as a result of a ±5% change in each property.

3.1 Young's moduli (E_{11} and E_{22})

The first property to be considered is the longitudinal Young's modulus (E_{11}). Figure 3 is a graph of percentage change in major curvature (Δa) as a function of edge length to thickness ratio (L/t) and percentage change in Young's modulus (ΔE_{11}). The figure shows that the maximum change in major curvature occurs when the edge length to thickness ratio is between 80 and 85, coinciding with the bifurcation point (see Fig. 2). This sharp peak in the change in curvature is caused by the variation in the value of E_{11} moving the bifurcation point of the laminate. Therefore there is a narrow region in the graph where the curvature of saddle shaped monostable laminates that have not reached the bifurcation point are compared to the curvature of cylindrical shaped bistable laminates. The large difference in the curvature of these different laminate shapes is responsible for the sharp peak in the change in curvature around the bifurcation point. A similar peak in the change in curvature around the bifurcation point is observed in all subsequent models for similar reasons. Beyond the bifurcation point and in the bistable region the percentage change in curvature converges to a limit of approximately 2.16% and -2.06% for a ±5% variation in E_{11}. A larger variation of approximately 2.38% to -2.28% is seen before the bifurcation point but in this region the laminate has a monostable saddle shape.

The sensitivity of the laminate to the transverse Young's modulus (E_{22}), Fig. 4, has a similar magnitude to E_{11} with the percentage change in curvature in the bistable region converging to values of -2.16% and 2.05% for a ±5% variation in E_{22}. While the magnitude of the variation in curvature for a ±5% change in the Young's moduli are similar the relationship is reversed; an increase in E_{11} decreases the curvature of the laminate, while the laminates curvature increases when E_{22} is increased. These results show that variations in E_{11} and E_{22} both have a significant effect on laminate shape.

Uncertainties in the Young's moduli originate from discrepancies between the real and quoted manufacturer values, which can be due to pre-peg decay during storage [27], variations in fibre volume fraction or errors in mechanical property measurement. However the Young's moduli can be relatively easily characterised post manufacture from simple stress-strain mechanical testing. Therefore while uncertainties in the Young's moduli can have a significant effect on laminate shape the difference between modelled and real values can be smaller than the 5% variation simulated here if accurate characterisation is undertaken.

The sensitivity of the laminate shape to ±5% changes in E_{11} and E_{22} for stacking sequences other than $[0/90]_T$ is considered by repeating the modelling process for additional cross-symmetric laminate layups of $[-15/75]_T$, $[-30/60]_T$ and $[-45/45]_T$. As each of these laminates has a different orientation of the major curvature the change in the maximum displacement of the laminate is plotted since it is directly related to the change in curvature. Figures 5 and 6 plot the percentage change in maximum displacement for a +5% change in E_{11} and E_{22} respectively. Both graphs show that for edge length to thickness ratios away from the bifurcation point the change in maximum displacement for all the layups tends to the same value. Near the bifurcation point a variation in the results is observed, primarily due to a variation in the location of the bifurcation point for each laminate. Since in all cases the change in maximum displacement converges to a very similar value as L/t increases, it is assumed that the sensitivity found for the $[0/90]_T$ cross-ply laminate applies to all cross-symmetric laminates. This has been confirmed by repeating this process for all subsequent properties, with the results not presented here for the purpose of brevity.

3.2 Poisson's ratio (v_{12}) and Shear modulus (G_{12})

Sensitivity analysis (See on-line Appendix A for figures) shows that the effect of variations in Poisson's ratio and shear modulus on bistable laminate shapes is almost negligible. Away from the peak at the bifurcation point a ±5% variation in v_{12} only causes a ±0.27% change in major curvature in the bistable region. Similarly away from the bifurcation point the variation in major curvature with a ±5% variation in G_{12} converges towards zero.

As the Poisson's ratio is particularly difficult to characterise experimentally [26], it is beneficial to note that large uncertainty in this property will not significantly affect the accuracy of the analytical model when away from the bifurcation point of the bistable laminate.

3.3 Thermal expansion coefficients (α_{11} and α_{22})

The sensitivity of the laminate to the longitudinal thermal expansion coefficient (α_{11}), shown in Fig. 7, is relatively small. The percentage change in curvature converges to a value of only ±0.52% for a ±5% variation in α_{11}. In contrast the transverse thermal expansion coefficient (α_{22}) has the greatest influence on curvature of all the material properties. In the bistable region the change in curvature converges to a value of ±5.55% for a ±5% variation in α_{22} as shown in Fig. 8. This is because, as can be seen in Table 1, α_{22} is an order of magnitude greater than α_{11}, and hence it is the principle property defining the magnitude of the thermal strains and the resulting curvature induced by cooling from the cure to ambient temperature. It is interesting to note that the effects of the longitudinal (fibre) and transverse (matrix) properties on the laminate shape are similar for both the Young's moduli (Figs. 3 and 4) and thermal expansion coefficients (Figs. 9 and 10). Increases in both the fibre dominated properties E_{11} and α_{11} decrease the curvature while increases in the matrix dominated values E_{22} and α_{22} increase the curvature, although the magnitude of this change is different.

As observed for the Young's moduli, uncertainty in the transverse thermal expansion coefficient can have a significant effect on laminate shape. While α_{22} may be more challenging to measure experimentally than the Young's modulus the thermal expansion coefficient can potentially be characterised to a high degree of accuracy [31]. With careful characterisation of the thermal expansion coefficients the discrepancy between the modelled and real laminate thermal expansion coefficient can be lower than the ±5% examined here.

3.4 Ply thickness (t)

Sensitivity analysis shows that uncertainty in the ply thickness can have a significant effect on the accuracy on an analytical model and the trend is similar to that of Young's modulus (See on-line Appendix A for figure). In the bistable region, away from the bifurcation point, the variation in curvature converges to 5.32% and -4.82% for a ±5% variation in ply thickness. Accurate characterisation of ply thickness depends on two factors, the thickness of the original pre-preg material and the change in ply thickness induced during manufacturing. The original pre-preg material thickness can be accurately measured however during manufacture there is a reduction in ply thickness due to compression during lay-up of the laminate. Ply thickness after cure can be simply measured using optical microscopy [16]. Provided that the manufacturing process used is suitably repeatable, little variation in ply thickness is expected between laminates and the final ply thickness should be predictable. It is therefore expected that the relatively high variations in laminate shape with ply thickness can be negated by post-cure characterisation and manufacturing quality control.

3.5 Moisture content (m)

It is typical to model zero moisture content in analytical models, thus we consider a range of moisture content (m) of 0 to 0.6%wt of laminate weight. We choose this range as the difference between the shape of a dry laminate and a laminate with 0.6%wt moisture content has previously been studied by Portela et al. [14]. The major curvature values calculated over this range are compared to the zero moisture content to consider the effect of moisture absorption on the laminate shape. As expected increasing the moisture content reduces the curvature as it relaxes the thermal residual strains. However the variation is very large with an 89.7% reduction in curvature for 0.6 %wt moisture content (See on-line Appendix A for figure).

While this shows that moisture content can have a significant effect on laminate shape it is important to consider the likelihood of such a high moisture content occurring. According to Etches et al. [15] a 0.6%wt moisture content is the maximum moisture content a laminate could absorb in a 35% relative humidity environment. The work of Portela et al. [14] further shows that to reach this maximum moisture content a T300/5028 graphite-epoxy composite would require 70 days of exposure to 35% humidity. Such conditions are unlikely to occur under standard laboratory conditions so a significant variation in curvature due to moisture would not be expected in experimental tests. However it is clear in any application that involves a prolonged exposure to humid conditions moisture absorption would cause significant changes in the laminate shape.

3.6 Cure and ambient temperature (ΔT)

Sensitivity analysis shows that accurate values of the change in temperature from cure (ΔT) are vital for accurate modelling of the laminate shape (See on-line Appendix A for figure). In the bistable region the percentage change in curvature converges to a significant variation of ±5.03% for a ±5% change in the temperature change from cure. The change in temperature from cure is dependent on both the ambient and cure temperatures. In this investigation the cure temperature is considered to be 180°C, a value that can be controlled to a high degree of accuracy during the manufacturing process and little variation is expected. The ambient temperature may be more difficult to control. The worst case considered in this study (±5% error) corresponds to approximately an 8°C variation in temperature. While the ambient temperature can clearly be controlled well within this limit during experimentation this result demonstrates that variation in operating conditions must be considered when modelling real world applications.

A summary of the sensitivities of the laminate curvature to ±5% change in each of the properties is shown in Table 2. Plots of the results not contained in the paper are provided as supplementary materials in the online edition of the manuscript in Appendix A. From these results it is clear that variations in material properties and environmental conditions can have a significant effect on laminate shape. The uncertainties can be a major source of discrepancies between analytical models and the experimental results. The model is observed to be most sensitive to the Young's moduli (E_{11} and E_{22}), transverse thermal expansion coefficient (α_{22}), ply thickness (t) and temperature change from cure (ΔT). Special care should be taken in characterising these critical parameters to minimise the uncertainties and improve confidence in the model prediction.

4. Inclusion of temperature dependent material properties

The previous section has identified that accurate characterisation of laminate properties, dimensions and ambient conditions can minimise errors between analytical models and experimental measurements of bistable laminates in laboratory conditions. However, in practical applications it is reasonable to expect large variations in the operating temperature. Therefore in this section the effects of large temperature change and the subsequent changes in material properties with temperature (Table 1) are considered. The coupled effect of using temperature dependent material properties with a ±5% change in temperature from cure to room temperature (21°C) will be initially considered; corresponding to a 13.05°C to 28.95°C ambient temperature range. The effect of the temperature dependency of each material property on laminate curvatures will be considered to identify which have the largest effect on laminate shape.

In the first instance we simply consider how each property varies with temperature. The change in the material properties over the 13.05°C to 28.95°C temperature range is shown in Fig. 9. This temperature change induces a less than ±5% change in E_{11}, v_{12} and G_{12}, with a maximum variation of -0.19% to 0.24% in E_{11}, -2.64% to 2.66% in υ_{12} and -1.01% to 0.95% change in G_{12}. The effect of temperature on E_{22} is larger with a change of -7.91% to 9.21%. The largest change occurs in α_{11} with a -44.44% to 50.69% change over this range of temperatures. Clearly E_{22} and α_{11} are the properties most sensitive to temperature for this particular material system.

A significant change in the sensitivity of the laminate shape to ±5% variation in the temperature change from cure is observed when the temperature dependent variation of all the material properties is included in the model in addition to the change in the thermal strains (See on-line Appendix B for the further details). The percentage change in curvature converges to values of 3.04% and -4.09%. This is lower than the change in curvature calculated when the material properties were kept constant at their room temperature values in section 3. The cause of this reduction in the variation in the laminate shape with temperature change is a result of the conflicting effects of the thermal strains and the changing material properties.

Examining the influence of the temperature dependence of each material property on the laminate curvature individually reveals that properties E_{11}, v_{12} and G_{12} have little influence on the laminate shape over this temperature range, inducing less than ±0.10%, ±0.14% and ±0.00% change in curvature respectively. This is unsurprising as Section 3 showed that laminate curvature is insensitive to variations of v_{12} or G_{12} and the temperature dependent variations of these material over the range of temperatures is small (Fig. 9).

Since the laminate was more sensitive to E_{22}, (Section 3) the large variation in E_{22} with temperature (Fig. 9) means that it has a significant effect, inducing a 3.66% to -3.51% variations in the curvature. The temperature dependent material property that has the greatest effect on laminate shape is α_{11}. While Section 3 showed that a ±5% variation in α_{11} had a relatively small effect on laminate shape compared to other material properties, the variation of α_{11} over the temperature range is considerably greater than any other material property (see Fig. 9). This large variation in α_{11} causes a total variation in the laminate curvature of -5.29% to 4.63%, the greatest effect of all the individual properties on the laminate curvature over this temperature range.

The changes in $E_{11,}E_{22,}$ v_{12} and G_{12} with decreasing temperature act to increase the laminate curvature as temperature reduces, complimenting the effect of the thermal strains. However The increase in α_{11}, as temperature decreases, reduces the curvature of the laminate, acting in opposition to the thermal strains. The conflicting effect of α_{11} on the laminate shape is responsible for the reduction in the change of laminate curvature when the temperature dependence of the material properties is included in the model. Table 3 summarises all the data showing α_{11} and E_{22} are the most important parameters.

It is clear from these results that ambient temperature variation has a significant effect on bistable laminate shape through the thermal strains and temperature dependent materials properties, even for a 5% (13.05°C to 28.95°C) variation in the difference between ambient room temperature and a 180°C cure temperature. However in practical applications a bistable laminate is likely to be subjected to larger variations in ambient temperature. The effect of subjecting the laminate to a typical aircraft operating range of 40°C to -60°C, is now considered. The variation in curvature across this temperature range is compared to that at room temperature (21°C).

With a temperature range of 100°C (40°C to -60°C) a very large change in laminate shape occurs when using constant room temperature material properties in the modelling, the relationship between curvature and temperature is linear, as observed by Giddings et al. [16]. Fig. 10 shows a change in temperature from 40°C to -60°C results in a large variation in curvature from the room temperature value of -12.02% to 51.19% respectively. When the temperature dependent material properties are considered along with the thermal strains the relationship between temperature and curvature becomes non-linear. As expected the increase in temperature from 21°C to 40°C reduces the laminate curvature by 11.08%. In addition, the curvature of the laminate initially increases as the temperature reduces from 21°C. However the curvature peaks at -1°C, converging in the bistable region to a maximum increase of 5.28%. Counter-intuitively the laminate curvature then begins to decrease with decreasing temperature, resulting in a significant reduction in curvature (-28.13%) at -60°C. Numerically this non-linear behaviour can be explained by the non-linear increase in α_{11} as temperature reduces; eventually α_{11} becomes the dominant laminate property that is changing with temperature, resulting in the in the prediction of a reduction in curvature of the laminate.

This relationship between the laminate properties and temperature increases the complexity of analytical modelling to design bistable laminates. Other composite systems with different material properties are likely to respond differently to temperature. For example, for the T300/5028 graphite-epoxy considered here α_{22} is larger than α_{11} by a single order of magnitude, however, for a M21/T800 prepreg α_{22} is over two orders of magnitude greater than α_{11} [27]. Preliminary sensitivity analysis of a $[0/90]_T$ M21/T800 laminate shows that in this case α_{11} has a much smaller effect on laminate shape than for T300/5028 graphite-epoxy, due to the greater α_{22}. If negligible variation of α_{22} with temperature is also observed for the M21/T800 composite then the influence of temperatures dependent material properties on laminate shape would be much smaller than for T300/5028 graphite-epoxy investigated here. However experimental data for the variation of the material properties of M21/T800 prepreg with temperature is unavailable to us at the time of writing.

5. Conclusions

Sensitivity analysis was performed on analytical models of bistable composite laminates to establish the influence of material, geometric and environmental uncertainties on laminate curvatures. The results reveal significant sensitivities of the laminate to Young's moduli (E_{11} and E_{22}), the transverse thermal expansion coefficients (α_{22}), ply thickness (t) and the temperature change from cure (ΔT). The difference between the manufacturers' values of these properties used in analytical models and the actual properties of the laminates can be a source of a significant proportion of discrepancies between the analytical model and experiments. It is concluded that significant improvements in the accuracy of bistable laminate shape predictions can be achieved if the laminate material properties, structure and operating environment are accurately characterised. Hence analytical modelling can be considered suitable for bistable laminate design when combined with accurate characterisation of material properties and a good level of quality control in the manufacturing process.

The significance of the temperature dependency of material properties was also evaluated. When temperature dependent materials properties were included in the model, there was a considerable increase in the curvature variation with temperature; it also significantly altered the relationship between the laminate shape and temperature. The analytical model predicted a counterintuitive reduction in the curvature of the laminate with decreasing temperature. This is opposite to what is predicted when constant (room temperature) materials properties are used. Further investigation including experimentation is necessary to validate this result and understand this counterintuitive behaviour over this range of temperature (40°C to -60°C). The results presented in this paper show that it is important to consider the temperature dependent material properties carefully when modelling bistable laminate behaviour, particularly for applications where a significant range of operating temperature is expected. This information is of importance for the modelling, design and materials selection of devices or structures based on bistable laminates.

Acknowledgments

The authors would like to thank the University of Bath EPSRC Enterprise Development Fund (EDF) for supporting some of this work.

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Figure Captions

Figure 1: The stable (solid line) and unstable shapes (dashed line) of a $[0/90]_T$ laminate with changes in edge length to thickness ratio. Point B is the critical bifurcation ratio.

Figure 2: Change in *x*-curvature of a $[0/90]_T$ laminate with increasing edge length to thickness (L/t) ratio.

Figure 3: Laminate sensitivity to 5% variation in longitudinal Young's modulus (E_{11}). Room temperature material properties.

Figure 4: Laminate sensitivity to 5% variation in transverse Young's modulus (E_{22}). Room temperature material properties.

Figure 5: Percentage change in maximum displacement with 5% change in E_{11} for four cross-symmetric layups of $[0/90]_T$, $[-15/75]_T$, $[-30/60]_T$ and $[-45/45]_T$.

Figure 6: Percentage change in maximum displacement with 5% change in E_{22} for four cross-symmetric layups of $[0/90]_T$, $[-15/75]_T$, $[-30/60]_T$ and $[-45/45]_T$.

Figure 7: Laminate sensitivity to 5% variation in longitudinal thermal expansion coefficient (α_{11}). Room temperature material properties.

Figure 8: Laminate sensitivity to 5% variation in transverse thermal expansion coefficient (α_{22}). Room temperature material properties.

Figure 9: The % change in material properties with variation in ambient temperature from 13.05°C to 28.95°C.

Figure 10: Laminate sensitivity to a change in temperature from 21°C to a 40°C to -60°C range, room temperature data only.

Figure 11: Laminate sensitivity to a change in temperature from 21°C to a 40°C to -60°C range, temperature dependent material properties.

Table 1: Material properties at room temperature and variation with temperature.

Table 2: Maximum change in curvature for a ±5% variation in each design variable as L/t increases.

Table 3: Converged change in curvature for a ±5% change in temperature change from cure (13.05°C to 28.95°C range), including temperature dependent material properties.